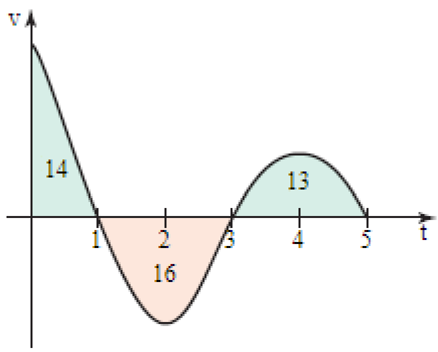
Calculus II, Final Exam Review

Please keep in mind that this is a general review of topics to study for your final. It is not meant to be all-inclusive.

- Chapter 6 -

1. Consider the graph shown in the figure, which gives the velocity of an object moving along a line. Assume time is measured in hours and distance is measured in miles. The areas of three regions bounded by the velocity curve and the t-axis are also given. Complete parts a – e.



a. On what interval(s) is the object moving in the positive direction?

b. What is the displacement of the object over the interval [0, 3]?

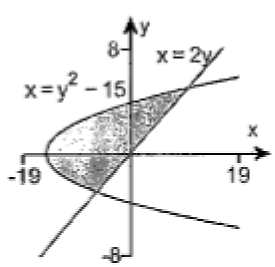
c. What is the total distance traveled by the object over the interval [0, 5]?

d. What is the displacement of the object over the interval [0,5]?

e. Describe the position of the object relative to its initial position after 5 hours.

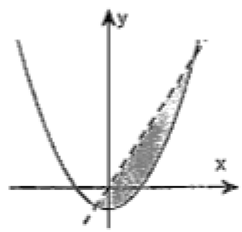
2. The function $v(t) = t^3 - 8t^2 + 15t$ on the interval [0, 7] is the velocity in m/sec of a particle moving along the x-axis. Complete parts a-c.
   a. Determine when the motion is in the positive direction and when it is in the negative direction.
   b. Find the displacement over the given interval
   c. Find the distance traveled over the given interval.

3. Determine the area of the shaded region in the given figure.



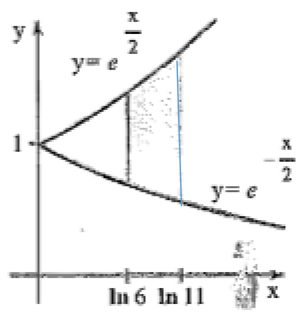
4. Find the area of the region bounded by $y = e^x, y = e^{-2x}, and\ x = ln3$
   Give an exact answer.

5. Determine the area of the shaded region below.
   The region is bounded by $y = x^2 - 7\ and\ \ y = 6x$. Give an exact answer

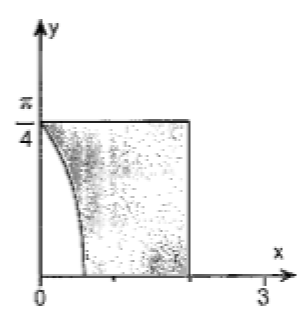


6. Let R be the region bounded by the graphs of $y = 5 - 2x, y = 0, and\ x = 0$. Find the volume of the solid generated by revolving the region about the x-axis. (Give an exact answer)

7. Let R be the region bounded by the following curves. Find the volume of the solid generated when R is revolved about the x-axis.
   $y = e^{\frac{x}{2}}, y = e^{\frac{-x}{2}}, x = ln6\ and\ x = ln11$ Give an exact answer



8. Find the volume of the solid generated when R (shaded region) is revolved about the given line.
   $y = 2 - \sqrt{2}\,secx, x = 2, y = \frac{\pi}{4}\ and\ y = 0; about\ x = 2$



9. Let R be the region bounded by the following curves. Use the shell method to find the volume of the solid generated when R is revolved about the x-axis. (Give an exact answer)
$y = 19 - x, y = x, y = 0$

10. Let R be the region bounded by the following curve. Find the volume of the solid generated when R is revolved about the y-axis. (exact answer)
$y = \frac{e^x}{4x}, y = 0, x = 1, and\ x = 5$

11. Let R be the region bounded by the curves y = 15x, y=15, and x=0. Find the volume of the solid generated when R is revolved about the y-axis.

12. Let R be the region bounded by $y = x^2, x = 1, and\ y = 0$. Find the volume of the solid generated when R is revolved about the line x=13

13. Use both washer and shell methods to find the volume of the solid that is generated when the region in the first quadrant bounded by $y = x^2$, y=25, and x=0 is revolved about the line x=t.
    a. Set up the integral that gives the volume using disk/washer method
    b. Set up the integral that gives the volume using shell method
    c. Volume =_______(Give an exact answer)

14. Let R be the region bounded by the curves $y = 3x^2, and\ y = 4 - x^2$. Use the method of your choice to find the volume of the solid generated when the region is revolved about the x-axis.

15. Let R be the region bounded by the following curves. Use the method of your choice to find the volume generated when R is revolved about the y-axis. $y = x, y = 3x + 3, x = 2, and\ x = 6.$

16. Find the arc length of the curve below on the given interval
$y = \frac{3}{4}x^{4/3} - \frac{3}{8}x^{2/3} + 9$ on [1, 27]

17. Find the area of the surface generated when the given curve is revolved about the given axis.

$y = \frac{x^3}{17}$ for $0 \leq x \leq \sqrt{17}$

- Chapter 7 -

18. Suppose a force of 60N is required to stretch and hold a spring 0.1m from its equilibrium position.

a. Assuming the spring obeys Hooke's law, find the spring constant k.

b. How much work is required to compress the spring 0.2 m from its equilibrium position?

c. how much work is required to stretch the spring 0.5 m from its equilibrium position?

d. How much additional work is required to stretch the spring 0.1 m if it has already been stretched 0.1 m from equilibrium?

19. A swimming pool has the shape of a box with a base that measures 24m by 18 m and a uniform depth of 2.9m How much work is required to pump the water out of the pool when it is full? Use 1000 kg/m^3 for the density of water and 9.8 m/s^2 for the acceleration due to gravity.

20. A cylindrical tank has height 8 m and radius 4m.

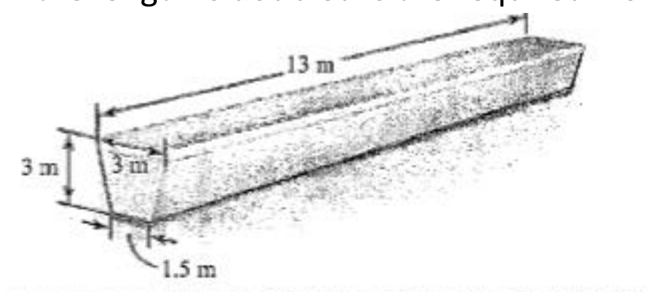
a. If the tank if sull of water, how much work is required to pump the water to the level of the top of the tank? Use 1000 kg/m^3 for the density of water and 9.8 m/s^2 for the acceleration due to gravity.

b. Is it true that it takes half as much work to pump the water out of the tank when it is half-full as when it is full? Explain.

21. A trough has a trapezoidal cross section with a height of 3 m and horizontal sides of width 3/2 m and 3 m. Assume the length of the trough is 13 m. See the figure below. Complete parts a and b.

a. How much work is required to pump the water out of the trough (to the level of the top of the trough) when it is full? Use 1000 kg/m³ for the density of water and 9.8 m/s² for the acceleration due to gravity.

b. If the length is doubled is the required work doubled?



22. Suppose that \$4000 is deposited in a savings account that increases exponentially. Determine the APY if the account increases to \$4800 in 4 years. Assume the interest rate remains constant and no additional deposits or withdrawals are made.

23. Uranium 238 has a half-life of 4.5 billion years. Geologists find a rock containing a mixture of U-238 and lead and they determine that 85% of the original U-238 remains; the other 15% has decayed into lead. How old is the rock?

- Chapter 8 -

24. Integrate: $\int \frac{x+8}{x^2+25}\,dx$

25. Integrate:

$$\int \frac{4-32x}{\sqrt{9-16x^2}}\,dx$$

26. Integrate: $\int \frac{1}{x^2+2x+26}\,dx$

27. Integrate: $\int 6\theta sec^2\theta d\theta$

28. Integrate: $\int 18x^2 lnx dx$

29. Integrate: $\int 16x csc^{-1}x dx \quad x \geq 1$

30. Integrate: $\int 8te^t dt$

31. Integrate: $\int t^3 e^{4t} dt$

32. Integrate: $\int \frac{4x^2+x+4}{(x-1)(x^2+x+1)} dx$

33. Integrate: $\int \frac{x^2}{(100-x^2)^{3/2}} dx$

34. Integrate: $\int \frac{dx}{(1+36x^2)^{3/2}}$

35. Integrate: $\int \frac{9}{x(x+3)} dx$

36. Integrate: $\int \frac{10x^2}{x^4-1} dx$

37. Integrate: $\int e^{-6\theta} sin6\theta d\theta$

38. Integrate: $\int 4sin^3x cos^2 x dx$

39. Integrate: $\int 16sin^2x cos^2 x dx$

40. Integrate: : $\int 7sec^3x tan\, x dx$

41. Integrate: $\int_2^{\infty} \frac{13}{xlnx} dx$

42. Integrate: $\int_2^3 \frac{1}{(x-2)^{3/2}} dx$

- Chapter 10 -

43. Write the first four terms of the sequence $\left\{\frac{1}{6^n}\right\}$

44. Find the limit of the sequence $\left\{\frac{2n^3-1}{3n^3+1}\right\}$

45. Find the limit of the following sequence or determine that the sequence diverges. $\left\{\frac{n}{e^n+20n}\right\}$

46. Evaluate the geometric series or state that it diverges

$$\sum_{n=0}^{\infty} e^{-4n}$$

47. For the following telescoping series, find a formula for the nth partial sum and then use that to evaluate the sum of the series or state that the series diverges.

$$\sum_{k=1}^{\infty}\left(\frac{2}{\sqrt{k+2}}-\frac{2}{\sqrt{k+4}}\right)$$

48. Evaluate the following series or state that it diverges

$$\sum_{k=1}^{\infty}\left(3\left(\frac{1}{4}\right)^k-2\left(\frac{3}{5}\right)^k\right)$$

49. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty}\frac{10k^6+k}{8k^6-12}$$

50. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty}\frac{(-17)^k}{k!}$$

51. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} \frac{(-1)^k k}{3k^5 + 1}$$

52. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} \frac{k^{10}}{10^k}$$

53. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} \frac{6}{(k+2)^5}$$

54. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} \frac{8 + cos 10k}{k^8}$$

55. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} (-1)^k \left(\frac{6k}{5k+8}\right)^k$$

56. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} k^7 e^{-3k}$$

57. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} \frac{2(3k)!}{(k!)^3}$$

58. Determine whether the following series converges. Justify your answer.

$$\sum_{k=1}^{\infty} cos\frac{13}{k^7}$$

59. Determine if the series converges absolutely, converges conditionally, or diverges

$$a.\ \sum_{k=1}^{\infty} \frac{(-1)^k}{2k+3} \qquad b.\ \sum_{k=1}^{\infty} \frac{(-1)^k tan^{-1}k}{k^3}$$

- Chapter 11 -

60. a. Find the linear approximating polynomial for the following function centered at a.

b. Find the quadratic approximating polynomial for the function centered ata.

c. Use the linear and quadratic polynomials to approximate the given quantity.

$$f(x) = 16x^{3/2}, a = 9;\ \ approximate\ 16(9.5)^{3/2}$$

61. a. Find the linear approximating polynomial for the following function centered at a.

b. Find the quadratic approximating polynomial for the function centered ata.

c. Use the linear and quadratic polynomials to approximate the given quantity.

$$f(x) = e^x, a = 0, approximate\ e^{0.8}$$

62. Determine the radius and interval of convergence of

$$\sum_{k=0}^{\infty}(19x)^k$$

63. Determine the radius and interval of convergence of

$$\sum_{k=0}^{\infty}(21kx)^k$$

64. Determine the radius and interval of convergence of

$$\sum_{k=0}^{\infty} sin^k\left(\frac{3}{k}\right)x^k$$

65. Determine the radius and interval of convergence for:

$$x^3-\frac{x^5}{8}+\frac{x^7}{27}-\frac{x^9}{64}+\cdots$$

66. a. Find the first four nonzero terms fo the Maclaurin series for the given function.

b. Write the power series using summation notation

c. Determine the interval of convergence of the series.

$$f(x)=(5+x^2)^{-1}$$

67. a. Find the first four nonzero terms fo the Maclaurin series for the given function.

b. Write the power series using summation notation

c. Determine the interval of convergence of the series.

$f(x) = 9sin3x$

68. Use the definition to find a Maclauren series for f(x)=ln(1+x), Use that series to find a Maclauren series for g(x)=ln(1+10x)

Chapter 12

69. Find a set of parametric equations for the parabola $y = x^2$

70. Find parametric equations for the following curve. Include an interval for the parameter values. A circle centered at (-4, -5) with radius 7, generated counterclockwise.

71. Find a parametric description of the line segment from the point P to the point Q. P(0,0), Q (7, -13).

72. Find parametric equations for the following curve. Include an interval for the parameter values.

The path consisting of the line segment from (-2, -7) to (0, -9), followed by the segment of the parabola $y = -9 + x^2$ from (0,0) to (3,0) using parameter values $-2 \leq t \leq 3$

73. Consider the following parametric equations, x= − 5t. y = 9t−11,

$-10 \le t \le 10$

a. Make a brief table of values for t, x, and y

b. Plot the points and complete the curve indicating positive orientation with arrows

c. Eliminate the parameter to obtain an equation in x and y

d. Describe the curve

74. Consider the parametric equations. $x = \sqrt{t} + 6, y = 5\sqrt{t}\,; 0 \le t \le 16$

a. Eliminate the parameter to obtain an equation in x and y

b. Describe the curve and indicated positive orientation.

75. Consider the parametric equations.

$x = 13cost, y = 1 + 13sint; 0 \le t \le 2\pi$

a. Eliminate the parameter to obtain an equation in x and y

b. Describe the curve and indicated positive orientation.

76. The polar coordinates of a point are given. Find the rectangular coordinates of the point. $(1, \frac{5\pi}{4})$

77. Express the Cartesian coordinates $(2, 2\sqrt{3})$ in polar coordinates in at least two different ways, one with the angle between 0 and 2π, the other with the angle between 0 and -2π

78. Find the slope of the line tangent to the polar curve at the given point.

$r = 9sin\theta; (\frac{-9}{2}, \frac{11\pi}{6})$

79. Make a sketch of the region and its bounding curves. Find the area of the region. The region inside the limacon $r = 2 + cos\theta$ (Give an exact answer for the area)

80. Make a sketch of the region and its bounding curves. Find the area of the region.

The region inside one leaf of $r = 2cos5\theta$

81. Find the area of the following region.

The region outside the circle r=3 and inside the circle $r = -6sin\theta$

82. find the area of the following region.

The region common to the circles $r = -4cos\theta \ and \ r = 2$

83. Find the area of the following region.

The region common to the circle r=8 and the cardioid $r = 8(1 - cos\theta)$

84. Determine the coordinates of the focus and the equation of the directrix then graph the equation $x^2 = -25y$

85. Sketch the graph of the parabola $y^2 = 16x$. Specify the location of the focus and the equation of the directrix.

86. Sketch the graph of the following ellipse. Plot the coordinates of the vertices and foci, and find the lengths of the major and minor axes. $\frac{x^2}{36} + y^2 = 1$

87. Sketch the graph of the following hyperbola. Specify the coordinates of the vertices and foci, and find the equations of the asymptotes. $\frac{y^2}{25} - \frac{x^2}{4} = 1$

88. Find the focus and directrix of the parabola with the equation $9x^2 + 10y = 0.$ $Then\ graph\ the\ parabola$

89. Sketch the graph of the following ellipse. Plot the coordinates of the vertices and foci, and find the lengths of the major and minor axes. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

90. Sketch the graph of the following hyperbola. Specify the coordinates of the vertices and foci and find the equations of the asymptotes.

$$\frac{x^2}{5} - \frac{y^2}{3} = 1$$