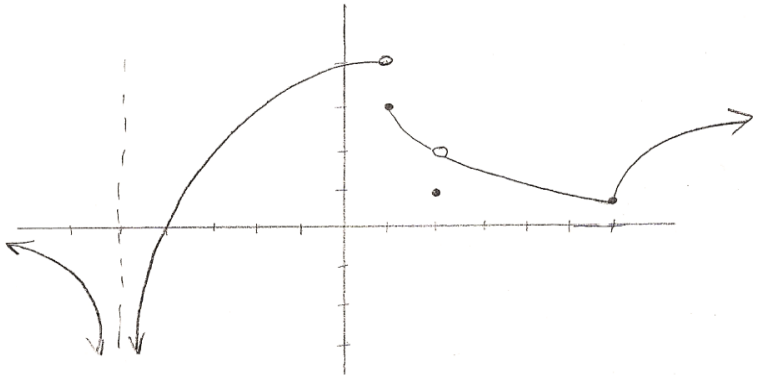
Always show all work and give exact answers unless otherwise specified.

1. Use the given graph to answer questions a – i



a. $\lim_{x\to 2} f(x) =$ b. $\lim_{x\to 1^+} f(x) =$ c. $\lim_{x\to 1^-} f(x) =$

d. $\lim_{x\to 1} f(x) =$ e. $\lim_{x\to -5^+} f(x) =$ f. $\lim_{x\to -5^-} f(x) =$

g. $\lim_{x\to -5} f(x) =$

h. List all x-values at which the function is not continuous. Label the discontinuities as removable or nonremovable.

i. List all the x-values at which the function is non-differentiable.

2. Evaluate the following limits analytically. Give exact answers.

a. $\lim_{x\to 0}(e^{-x})\sin \pi x$ b. $\lim_{x\to 0}\dfrac{\sqrt{x+2}-\sqrt{2}}{x}$

c. $\lim_{x\to 2}\dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$ d. $\lim_{x\to 0}\dfrac{2\sin x}{3x}$

e. $\lim_{x\to 2}(\ln 2x + e^{3x}+1)$ f. $\lim_{x\to 0}\dfrac{e^{2x}-1}{e^{x}-1}$

g. $\lim_{x\to 0}\dfrac{3^{2x}-1}{3^{x}-1}$ h. $\lim_{x\to 4}\dfrac{\sqrt{5-x}-1}{2-\sqrt{x}}$

i. $\lim_{t\to 0}\dfrac{\cos t-\cos^2 t}{t}$ j. $\lim_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$

3. Find the limits using any appropriate method.

a. $\lim_{x\to 3^+} \dfrac{|3-x|}{x-3}$

b. $\lim_{x\to 1} f(x) = \begin{cases} -2x+3, x<1 \\ x^2, x>1 \end{cases}$

4. Find the x-values (if any) at which *f* is not continuous. Identify any discontinuities as removable or non-removable. **(Section 2.4)**

a) $f(x) = \dfrac{x+2}{x^2-3x-10}$

b) $f(x) = \begin{cases} \ln(x+1), x \geq 0 \\ 1-x^2, x<0 \end{cases}$

5. Find any vertical asymptotes for a) $y = \frac{x-1}{x^2+1}$ b) $y = \dfrac{x-1}{x^2-1}$

Part II—Derivatives and Applications of Derivatives

6. Use the definition of derivative to find *f'(x)* for each of the following: **(Section 3.1)**

a. $f(x) = x^2 - 3x + 2$

b. $f(x) = \sqrt{3-x}$

c. $f(x) = \dfrac{2}{x+1}$

7. Find $\dfrac{dy}{dx}$ for each of the following: **(Sections 3.2 – 3.6)**

a) $y = x^3 \tan 2x$

b) $y = \sqrt{\cos\sqrt[3]{x}}$

c) $x\cos y = 1$

d) $y = \dfrac{x^2\sqrt{3x-5}}{\sqrt{2x+3}}$

e) $y = (1+x)^{\frac{2}{x}}$

f) $y = \arcsin t^2$

g) $y = e^x \arctan x$

h) $y = \tan(\arcsin t)$

i) $y = \sin(arc\sec x)$

j) $y = \left(\dfrac{x^2}{x^3-1}\right)^4$

k) $y = x^2\sqrt{14+2x^2}$

l) $y = e^x(\sin x + \ln x)$

m) *sinx=x(1+tany)*

$y = \sinh^3 x \cosh^4 3x$

8. For each of the following, find an equation of the tangent line to the graph of f at the given point.

   a. $f(x) = \dfrac{e^x}{(x+4)}$ (0, ¼)

   b. $f(x) = \dfrac{1}{\left(x^2 - 3x\right)^2}$ (4, 1/16)

Related Rates (Section 3.7)

9. The edges of a cube are expanding at a rate of 5 centimeters per second. How fast is the surface area of the cube changing when each edge is 4.5 centimeters?

10. A baseball diamond has the shape of a square with sides 90 feet long. A player is running from first second at a speed of 28 feet per second. Find the rate at which the distance from home plate is changing when the player is 30 feet from second base.

11. A person flying a kite holds the string 5 feet above ground level, and the string is payed out at a rate of 2 ft / sec as the kite moves horizontally at an altitude of 105 feet (see figure). Assuming there is no sag in the string, find the rate at which the kite is moving when 125 feet of string has been payed out.

12. A girl starts at a poin A and runs east at a rate of 10 ft/sec. One minute later, another girl starts at A and runs north at a rate of 8 ft/sec. At what rate is the distance between them changing 1 minute after the second girl starts?

13. An airplane at an altitude of 10,000 feet is flying at a constant speed on a line that will take it directly over an observer on the ground. If, at a given instant, the observer notes that the angle of elevation of the airplane is 60 degrees and is increasing at a rate of 1 degree per second, find the speed of the airplane.

14. A missile is fired vertically from a point that is 5 miles from a tracking station and at the same elevation. For the first 20 seconds of flight, its angle of elevation θ changes at a constant 2° per second. Find the velocity of the missile when the angle of elevation is 30°

15. A balloon rises at a rate of meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

16. A woman standing on a cliff is watching a motorboat through a telescope as the boat approaches the shoreline directly below her. If the telescope is 250 feet above the water level and if the boat is approaching at 20 feet per second, at what rate is the angle of the telescope changing when the boat is 250 feet from the shore?

17. A student is using a straw to drink from a conical paper cup, whose axis is vertical, at a rate of 3 cubic centimeters per second. If the height of the cup is 10 centimeters, and the diameter of its opening is 6 centimeters, how fast is the level of the liquid falling when the depth of the liquid is 5 centimeters?

18. Locate the absolute extrema of the function on the closed interval. **(Section 4.1)**
    a) $y = 3x^{2/3} - 2x$ [-1, 2]
    b) $f(x) = x\ln(x+3)$ [0, 3]
    c) $y = e^x \sin x$ $[0, \pi]$

19. Determine whether Rolle's Theorem can be applied to f on the closed interval [a,b]. if Rolle's Theorem can be applied, find all values of c in the open interval (a,b) such that $f'(c) = 0$. **(Section 4.2)**
    a) $f(x) = \dfrac{x^2 - 1}{x}$ [-1, 1] b) $f(x) = x - 2\ln x$ [1,3]

20. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a,b]. If the mean Value Theorem can be applied, find all values that satisfy the conclusion of the Mean Value Theorem. **(Section 4.2)**
    a) $f(x) = \sin x$ **on** $[0, \pi]$**.** b) $f(x) = e^{2x}$ [0,1]

21. Find the critical numbers (if any). Find the open intervals on which the function is increasing or decreasing and locate all extrema. **(Section 4.3)**
    a. $f(x) = x^3 - 6x^2 + 15$
    b. $f(x) = (x-1)^{1/3}$
    c. $f(x) = x\arctan x$
    d. $f(x) = \sin^2 x + \sin x$ on the interval $[0, 2\pi]$

22. Find the points of inflection and the open intervals on which the graph is concave up and concave down.
    a. $f(x) = 2x^4 - 8x + 3$
    b. $y = x - \ln x$

23. Find the limits**. (Section 4.5)**
    a. $\lim\limits_{x\to\infty} \dfrac{2x^2 + 5}{x^2 - 4}$ b. $\lim\limits_{x\to\infty} \left(2 - 5e^{-x}\right)$ c. $\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}}$
    d. $\lim\limits_{x\to-\infty} \dfrac{3x}{\sqrt{x^2 - 3x + 2}}$ e. $\lim\limits_{x\to\infty} \dfrac{2x+3}{\sqrt{x^2 - 3x + 2}}$

24. Section 4.6—A Summary of Curve Sketching

For each of the following find

i. domain
ii. x and y intercepts
iii. vertical and horizontal or oblique asymptotes
iv. first derivative, intervals where the graph is increasing or decreasing
v. relative extrema
vi. points of inflection
vii. intervals where the graph is concave up or down
viii. then graph by hand.

a) $f(x) = \dfrac{x^2+1}{x^2-9}$ b) $f(x) = x\sqrt{16-x^2}$ c) $f(x) = x^5 - 5x$

d) $f(x) = e^{3x}(2-x)$ e) $f(x) = \dfrac{x^3}{x^2-1}$

Section 4.7—Optimization Problems

25. You must make a small rectangular box with a volume of 400 in^3. Its bottom is a rectangle whose length is twice its width. The bottom costs 7 cents per square inch; the top and four sides of the box cost 5 cents pr square inch. What dimensions would minimize the cost of the box?
26. Each page of a book will contain 30 in^2 of print, and each page must have 2-in. margins at top and bottom and 1-in margins at each side. What is the minimum possible area of such a page?
27. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?
28. Find two positive numbers such that the sum of the first and twice the second is 100 and the product is a maximum.
29. A man is in a boat 2 miles from the nearest point on the coast. He is to go to a point Q, located 3 miles down the coast and 1 mile inland. He can row at 2 miles per hour and walk at 4 miles per hour. Toward what point on the coast should he row in order to reach point q in the least time?
30. A rectangle has two corners on the x-axis and the other two on the parabola $y = 12 - x^2$, with y ≥ 0. What are the dimensions of the rectangle of this type with maximum area.
31. Find the points on the parabola $x = 2y^2$ that are closest to the point (10,0).

PART 3—Integration and applications

Section 5.1—Antiderivatives and Indefinite integration

33. Integrate the following

a) $\int \frac{x^2+x+1}{\sqrt{x}}dx$ b) $\int (3x^3 - 5\sqrt[4]{x^3} - 2/x + 4x^{-5} + 2\sin x)dx$

c) $\int (\sec^2 x - \csc^2 x)dx$ d) $\int \frac{\sin^3 x}{1-\cos^2 x}dx$

34. Solve the differential equation; $f''(x) = x^2$, $f'(0) = 6$, $f(0) = 3$

35. Bill throws a stone upward from the ground. The stone reaches a maximum height of 225 ft. What was its initial velocity?

36. An object has a constant acceleration of -42 feet per second squared, an initial velocity of 5 feet per second, and an initial position of 9 feet. Find the position function describing the motion of this object.

37. Approximate the area below the graph of $f(x) = 4 - x^2$ and above the x-axis between x=0 and x=2. Use 4 rectangles and find both the upper and lower approximation. (**Section 5.2)**

38. Use the limit definition of area to find the actual area below $f(x) = 4 - x^2$ and above the x-axis between x =0 and x=2. **(5.2)**

39. Use the limit process to find the area of the region between the graph of the function $f(x) = 2x - x^2$ and the x-axis over the interval [0, 2].

40) Evaluate the definite integral by the limit definition $\int_{-1}^{2} (2x+5)dx$ **(Section 5.3)**

Integration techniques (Sections 5.4 – 5.9)

Evaluate the following definite integrals

41) $\int_{-1}^{1} (t^2+1)dt$ 42) $\int \frac{x}{\sqrt{x^2+3}}dx$ 43) $\int \cot^4 2x \csc^2 2x dx$

44) $2\pi \int_{-1}^{0} x^2\sqrt{x+1}dx$ 45) $\int_0^e \frac{\ln x}{x}dx$ 46) $\int \cos x \sin x \, dx$

47) $\int \frac{5x+16}{x^2+9}dx$ 48) $\int \frac{x-1}{\sqrt{x^2-2x-24}}dx$ 49) $\frac{d}{dx}\int_2^x \sqrt{4\sin t}dt$

50) $\int \frac{2x+3}{2x^2+6x-9}dx$ 51) $\int_{\pi}^{\pi} \sin^3 x \cos x dx$ 52) $\int \frac{e^x}{4+9e^{2x}}dx$

53) $\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$

54. Find the average value of $f(x) = 2x^2 + 3$ on the interval [2,4]

Think About it

55. Create a function that has a vertical asymptote at x=5 and a horizontal asymptote at y=2

56. Sketch a graph of a differentiable function f that satisfies the following conditions:
$f'(x) < 0$ when x<2 $f'(x) > 0$ when x>2
$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 6$

57. Sketch the graph of a function that does NOT have a point of inflection at (c, f©)even though f'’(c)=0

58. True or false: The graph of every cubic polynomial has precisely one point of inflection

59. True or false. If $f''(2) = 0$, then the graph of f must have a point of inflection at x=2